**CMPT 335 Discrete Structures**

Homework 7

1. Prove the following generalization of the De Morgan’s law using induction (*Aj* ⊆*U; j* =1,...,*n* )

Basis: =

Hypothesis: = = = …

Step:

= = = =

∩ ∩… ∩ ∩ =

1. Prove the following generalization of the De Morgan’s law using induction

= ∨… ∨

Basis: =

Hypothesis: = = ∨… ∨ = …

Step:

= = = =

∨ ∨… ∨ ∨ = ∨… ∨

1. Find a formula by examining the values of the expression 1/2 + 1/4 + 1/8 +… + 1/2^n and prove it using induction

**(2^(n) - 1 )/2^n**

Let P(n) be the statement 1/2 + 1/4 + 1/8 + ... + 1/2^(n) = (2^(n) - 1 )/2^n

Basis: When n = 1, 1/2 on the left-hand side equals 1/2 on the right-hand. Hence P(1) is true.

Step: Assume that P(n) is true.   
Then 1/2 + 1/4 + 1/8 + ... + 1/2^n + 1/2^(n+1)   
= (2^n - 1 )/2^n + 1/2^(n+1)  
= (2\*2^n - 2 + 1 )/2^(n+1)   
= (2^(n+1) - 1 )/2^(n+1)

Hence, P(n+1) holds true.

1. Prove the inequality 2*n* < *n*! for *n* ≥ 4,*n* ∈ **ℤ**

Test n=4:

2^4 < *4*!

16 < *24 ✓*

Assume true for n=k

2^k < *k*! {k≥ 4}

Prove true for n=k+1,

2^(k+1) < (*k+1)*!

(k+1)! – (2^(k+1)) > 0 Since this is true, I can conclude that (k+1)! is in fact greater than 2^(k+1).

1. Verify the correctness of the Selection Sort algorithm (ascending order) using induction.

Algorithm:

**for** j = 1 to **length** (A)-1

{

**for** i=j+1 to **length (**A)

**if** A[j]>A[i] // if this is true, swap A[i] and A[j]

{

key=A[j]

A[j]=A[i]

A[i]=key

}

}

Basis: Suppose an array consists of 2 elements. The key is A[j]. If A[j] is greater than A[i], swap A[i] and A[j], thus making the key A[i].

Hypothesis: Suppose this algorithm may sort an array containing n elements.

Step: We have an array containing n+1 elements. Suppose first n of them are already sorted (based on the inductive step). Let us consider the last iteration of the for loop, j=n-1. Then i=n.

The key is A[n-1]. If A[n-1] is greater than A[n], swap A[n] and A[n-1], thus making the key A[n]. Since first n elements are already sorted, this means that an entire array is already sorted.